Mikhail Feldman
Proposed Ph.D. Course Syllabus:
Free Boundary Problems for Elliptic Equations and Shock Analysis

Course Content.

The goal of this course is to present techniques used in the study of free boundary problems for elliptic equations. Basic questions are existence and regularity of solutions, and the properties of free boundaries, study of which is usually more subtle and relies on geometric and harmonic analysis techniques. Classical free boundary problems, and several problems arising in applications to shock analysis in compressible fluid dynamics will be discussed.

Topics include:

- Review of regularity theory for elliptic equations (2 hours)

- Introduction to classical free boundary problems: obstacle problem, one-phase problem, two-phase problem (1 hour)

- The obstacle problem: weak formulation, existence and regularity of solutions, regularity of the free boundary (4 hours)

- Introduction to regularity theory for one-phase problems (3 hours)

- Equations of compressible fluid dynamics: compressible Euler system, potential flow. Weak solutions, Rankine-Hugoniot conditions (2 hours)

- Free boundary problems in the study of steady shocks. (2 hours)

- Brief overview of self-similar shock reflection and free boundary problems in its analysis (2 hours)

Textbook: None required.

Some reference texts:

1. "Elliptic Partial Differential Equations" by D. Gilbarg and N. Trudinger

2. "Elliptic Partial Differential Equations" by Q. Han and F.H. Lin

3. "A Geometric Approach to Free Boundary Problems" by L. Caffarelli and S. Salsa

4. "Variational Principles and Free-Boundary Problems" by A. Friedman

5. "The Mathematics of Shock Reflection-Diffraction and von Neumann's Conjectures" by G.-Q. Chen and M. Feldman

Course Schedule.

The course will meet for 2 hours on Tuesdays and Thursdays (4 hours per week) over a 4-week period during my visit: June 1 – June 30, 2026.

Grading.

Grades will be based on two homework assignments: the first due at the midpoint of the course (after two weeks), and the second at the end of the course.